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DIFFERENT ANALYSIS STRATEGIES IN THE STRUCTURAL ASSESSMENT OF THE ST. ANNE OF PIETRA PERDUCA ORATORY

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Abstract. *The paper deals with some open issues about direct investigations and structural analysis related to the conservation of the architectural heritage. At first, it discusses the theme of the importance of the direct investigation on the monument, aimed to a full and conscious knowledge of the building. Then it focuses on structural analysis, which is based on the information obtained by means of surveys and it is not just an act for the project, but becomes itself an instrument of knowledge, that must be integrated with all other methods of investigation. The work focuses on the case study of the Oratory of St. Anne of Pietra Perduca, located in the Trebbia valley, close to Piacenza (Italy). The masonry of the building is made of marly limestone ashlars. These ashlars were laid with mortar placed into the wall's interstices and not visible on the external face, giving impression of dry masonry. The pitched roof has a timber structure with trusses and limestone roofing. Different structural analysis techniques were used for the structural assessment on the different building elements. For the timber roof structure linear static analysis was used, while for the masonry triumphal arch and the presbytery vault graphical methods, starting from Mery's and Heyman's formulations, were followed by a detailed numerical study by means a nonlinear macro-element non-linear modelling..*

1 ISSUES FOR THE CONSERVATION OF THE ST. ANNE ORATORY

The primary purpose of any restoration is the conservation, not only of the matter, as a testimony of the past, but mainly of the formal and historical value that an object conveys. The restoration of material decay and intervention on structural instability contribute to the maintenance of the integrity of the product. Therefore, it is essential to carry out a careful analysis of the building structure, which is necessarily constrained to the knowledge of specific data and information about building's materials, techniques, and health status. So the direct investigation on the structure is very important.

The first moment of direct knowledge is architectural surveying, as the critical tool that combines the geometric and morphological measurements to the information derived from the survey of materials and their degrade as well as information obtained by means of photogrammetric techniques [1].

The second important step is the historical investigation. It makes use of critical reading of the bibliographies of iconographies and of archival documents, properly integrated with the information derived from the survey and the physical-chemical properties. Historical documents can provide information about building materials and the physico-chemical add information about their characteristics. The integration of this different kind of knowledge obtained leads to a conscious and comprehensive understanding of the state of matter, of the temporal evolution sustained by the building and of structural instability on which you choose to intervene.

The cognitive method described above was applied to the conservation project of the Oratory of St. Anne of Pietra Perduca (Figure 1). This oratory is a little religious building of the 13th century made of stone masonry founded on high ophiolite in the Trebbia valley. As first instrument of knowledge, a geometrical and morphological survey campaign was conducted on the building.

The building has a rectangular single nave, which leads to the square-plan presbytery, which has a flat apse defined by a small niche in the wall. Near the presbytery there is the vestry, recently rebuilt in 2010 because of its high state of decay. The direct survey also gave information on the texture of walls: all the walls are made of the same kind of limestone and have a thickness ranging from 50 to 70 cm. The study of archival sources led to the determination of the construction phases of the whole oratory, starting from its foundation in the 13th century, through several expansions over the centuries, to the current conformation, with the presbytery, the sacristy and the bell tower [2].

Figure 1. Picture (left) and plan view (right) of the Oratory of St. Anne of Pietra Perduca

During the direct investigation it was not possible to implement physical-chemical or mechanical tests on building materials, so we used for comparison approach the results of the petrographic tests [3] made on mortar and units in the nearby village of Embresi [4]. According to literature studies about historical buildings and rural areas of the Trebbia valley both the oratory and the borough of Embresi belong to the same family of religious and civil buildings with specific characteristics of traditional construction techniques [5]. Similarities can be found in the single chamber structure, the use of marly limestone ashlars, the roof covered with slate and the corner solutions solved using bigger blocks then the ashlars of the stonework texture and laid across themselves [6]. For these reasons, proceeding by analogy with the buildings of Embresi, it was possible to deduce the most important data of composition and hardness of the stones.

The geometric, morphological, petrographic and historical data, with the observation of crack patterns and static instability, induced the analysis of the structural elements in order to define the conservation interventions.

2 STRUCTURAL ANALYSIS OF THE TIMBER ROOF

The structural analysis was carried out mainly on the roof elements and on the masonry ones, in particuar on the barrel vault and on the arch of the presbytery. It was performed according to different approaches in relation to the item investigated:

- linear static analysis for the roof;
- non-linear static analysis with simulation of the kinematics of the masonry portions.

For the definition of loads and material properties, reference was made to the Italian building code [7] and its commentary [8].

The structural system of the single nave roof is based on the scheme of non-thrusting roof, and it is made of three parallel wooden trusses, placed at about 3.5 meters each other. Three purlins are laid perpendicular to the trusses. One is placed on the ridge (ridge board) and the others placed in the middle of the pitches. Wooden rafters lay on the purlins perpendicular to the line of the ridge, spaced at distance varying between 60 and 85 cm. The roof covering is made of three layers of 4 cm thick slate slabs, placed with mortar on a wooden plank, laid on the rafters.

The load analysis provides a definition of the actual weights of the structural elements and the values of the loads imposed on it. Table 1 reports the considered acting loads.

Table 1: Loads on the roof.

Load	Description		Value
G_1	Weight of structural elements	rafters	108.9 N/m
		purlins	356.4 N/m
G_2	Weight of non- structural elements	roof covering	4455 N/m^2
Q	Accidental loads		500 N/m^2
N	Snow		1970 N/m^2
V	Wind	upwind pressure	171.88 N/m^2
		downwind pressure	312.5 N/m^2

The mechanical properties of the timber elements are:

- mean bending strength f_m = 42;
- mean shear strenght f_v = 4 MPa.

2.1 Rafters

The rafters are made of oak wood and have a rectangular cross-section of 9 x 11 cm^2. The distance between them is about 84 cm. Their static scheme is defined by an inclined continuous supported at three points.
The safety check performed on the timber rafters by using linear analysis results were all largely satisfied.

2.2 Purlins

Roof purlins are three, two of which placed symmetrically in the middle of the pitches, and one ridge board placed on the top. The static scheme of the purlin, tilted from the horizontal, requires that the calculation of all stresses must be the result of the analysis combination of perpendicular (y) and parallel loads (x) in relation to the element axis.

The static scheme adopted for the analysis of the purlins is reported in Figure 2.

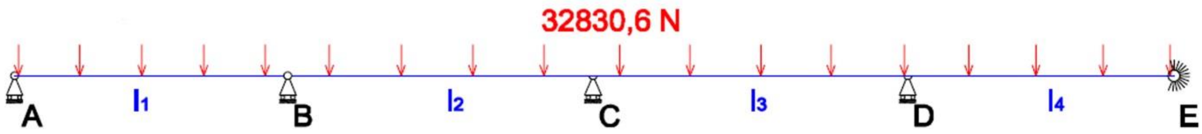


Figure 2: The static scheme of the purlin.

Table 2 reports a summary of the safety checks performed on the purlins.

Table 2: Purlins analysis: geometry, moment and shear stresses.

Rod	Section [cm]	L [cm]	Load [N/m]		M + [N·m]		M – [N·m]		V_{max} [N]	σ [MPa]	τ [MPa]
			x	y	x	y	x	y			
							0	0			
AB	18 x 18	313	2295	4706	28199	57817			73769	**86.78**	3.00
							0	0			
BC	18 x 21	349	1646	3375	18406	30358			72759	31.23	2.83
							23439	48056			
CD	18 x 21	360	1997	4094	10056	20619			74995	25.29	2.92
							21285	43641			
DE	18 x 21	305	1884	3863	12631	25899			73299	31.76	2.85
							0	0			

As it can be noticed from Table 2, only the flexural strength of the span AB is not verified. An intervention is hence necessary. Possible considered options consist in changing or strengthening the wooden beam, as well as modifying of the static scheme for the structural element.

2.3 The roof truss

The structure of the roof includes three timber trusses with similar arrangement. The loads are applied to the trusses through the support points of the purlins on the principal rafter. The central truss is considered because it is most loaded. It covers a 7 m span and is made of elements of variable section (22 x 23, 23 x 24, 18 x 21 and 13 x 13 cm^2 for the principal rafters, the chain, the monk and the diagonal posts, respectively). The results of linear analysis are reported in Figure 3.

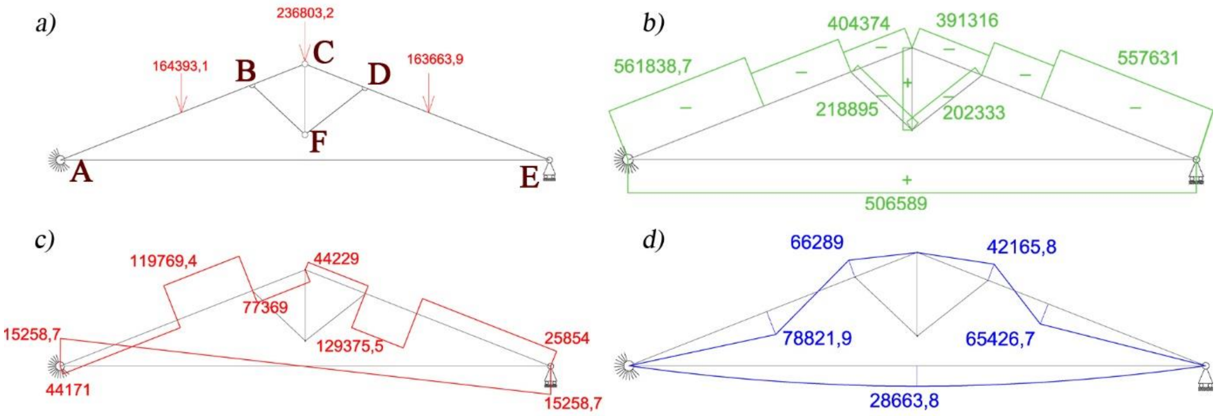


Figure 3: The roof truss. Static scheme *a)*; diagrams of axial force *b)*, shear force *c)* and bending moment *d)*.

In order to improve the flexural strength and stiffness of the principal rafters, the application of a post-tensioning intervention is suggested (Figure 6). Steel posts connected to the intrados of the rafters allow exploiting the deformation reduction effect of eccentrically tensioned steel cables [9].

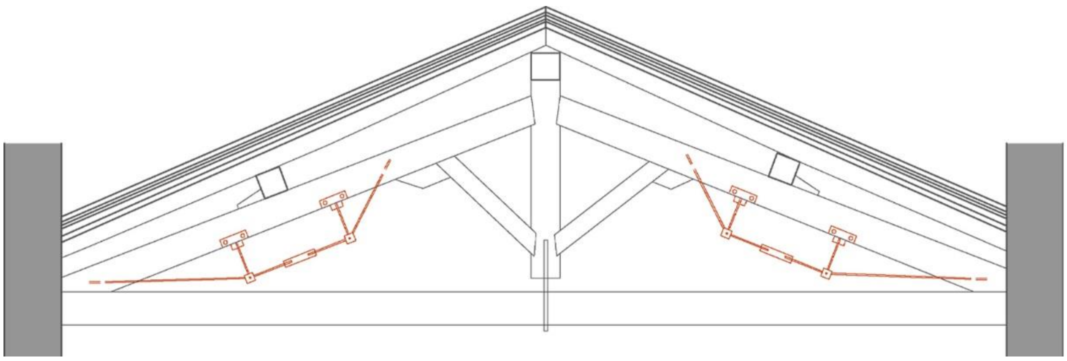


Figure 6: The post-tensioning intervention on the roof truss.

3 MASONRY STRUCTURE

The masonry of the oratory is identified as undressed stone masonry with facing walls of limited thickness and infill core (*sacco*). Marly limestone units, are assembled in two parallel leaves, occasionally interrupted by through stones. The mechanical material properties, used for the structural analyses, correspond to the minimum values of the ranges proposed in the Commentary to the Italian Building Code [8]. The following values are adopted:

- compressive strength f_m = 2.0 MPa;
- Young's modulus E = 1020 MPa;
- Shear modulus G = 340 MPa.

A stone density of 2200 kg/m^3 was measured directly on a sample, consistent, considering the presence of mortar and voids, with the value of 2100 kg/m^3 suggested in [8] for masonry density.

We used two different analytical methods for the different structural portions investigated. For the façade, which shows a significant vertical crack, the study was articulated according to the following steps:

1. identification and interpretation of the cracking pattern;
2. identification of the masonry portion potentially involved in a local out-of-plane failure mode (kinematic chain of rigid bodies);
3. computation of the lateral capacity by means of equilibrium limit analysis.

Concerning the masonry vault and the arch in the presbytery, the analysis relies on the TREMURI program [10], which carries out a macro-elements non-linear analysis considering the effects of lateral loads for increasing horizontal displacements (pushover analysis). Also in this case, several phases have been considered:

1. analysis based on graphic statics (method of Mery);
2. graphic statics verification with identification of the limit of the lateral load (Heyman's criterion);
3. numerical pushover analysis by means of macro-element models [11] also accounting for second order effects [12].

3.1 The masonry façade

A significant vertical crack covers the wall corner portion. This crack pattern suggests that the cohesion loss between the mortar and blocks is due to horizontal forces perpendicular to the western front, which also suffered some out-of-plane overturning. The wall corner portion vulnerable to out-of-plane overturning is reported in Figure 7. Figure 8 shows the activation scheme of the kinematic system which causes the overturning of the wall around the rotation point O at the ground level.

The weight P of the portion considered is 268130 N and the lengths b_l and b_p are respectively 0.26 m and 3.95 m.
Setting a simple rotation equilibrium around point O, we have the following values of stabilizing (MS) and overturning moment (MR):

$MR = P \cdot b_l = 69713.8$ Nm,

$MS = \alpha P \cdot b_p$.

The equilibrium is satisfied for $\alpha = 0.065$, calculated by equating the two expressions MR and MS. To improve this low value of the load multiplier, the proposed intervention includes the repair of the vertical crack and the insertion of a steel tie-rod in the top part of the wall (parallel to the overturing direction). On the outer surface of the western wall, the tie rod is constrained with a plate anchor.

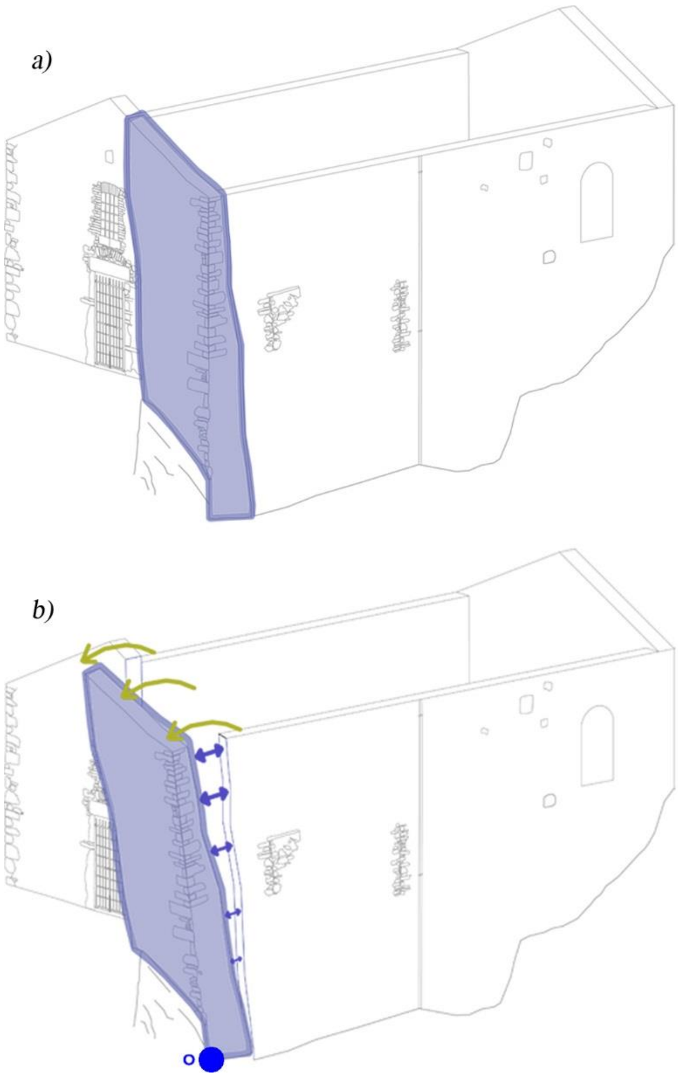


Figure 7: Front wall: kinematic chain *a)*; kinematic activation *b)*.

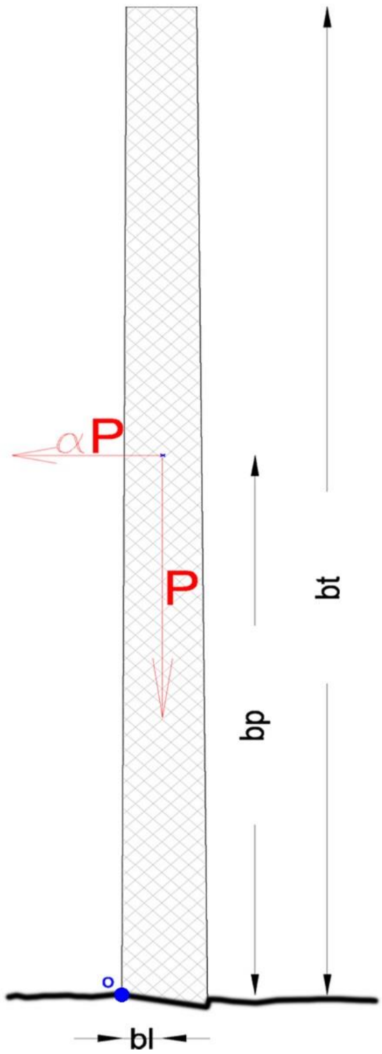


Figure 8: Wall section static scheme.

3.2 The masonry vault

The ceiling of the presbytery is internally formed by a masonry barrel vault. Outside the roof structure is organized with inclined pitches that directly load the perimeter wall and do not affect the vault.

Taking advantage of its symmetry, we built a model congruent to half vault. The model is divided into ten equal portions which are loaded with their own weight and with the possible external loads.

The weight of each masonry ashlar is 1299.4 N, which already accounts for the mortar weight, by means of a 20% increase. The volumic mass of generic filling is assumed equal to 1800 kg/m^3, which corresponds to a weight of 193.8 N. With the aid of the graphical method of Mery, assuming the tangent of the curve at the vault keystone, we derived the ideal thrust line which is always contained in the safety core of each ashlar. In this way we can define the vault thrust on the abutments. The vault system is safe, because there is no implication of tipping (Figure 9a) [13].

In order to assess the seismic response of this structural sub-system, the kinematic analysis studies the behavior of all components of the vault-abutments system subjected to a increasing horizontal thrust, proportional to the masses involved. We studied the whole system by applying horizontal forces whose magnitude is equal to the vertical forces value increased by the factor α. The vault is not in a static equilibrium when the magnitude of the horizontal forces induces the curve of pressures to touch or exceed the thickness of the vault itself. This comes from the Heyman’s safety criterion [14], which, differently to the Mery’s method, states that the vault is safe if the line pressure is all contained in its thickness (Figure 9 b,c). In the cases studied, the α limit value is 0,12. The displacement of an element also involves other effects, which change with the displacement. In order to consider these effects, we modelled the vault using the TREMURI program by assimilating ashlars to prismatic 2-node macro-elements (2D analysis).

Nodes 1 and 21 were modeled as polygonal, recreating the actual shape of the abutments. Below these nodes, two macro-elements serve as interface elements between the nodes 1 and 100 and between 21 and 101. Nodes 100 and 101 are fully restrained as they correspond to the points where the structure is connected to the rock. Vertical loads proportional to the considered masses are applied on the nodes (Figure 10).

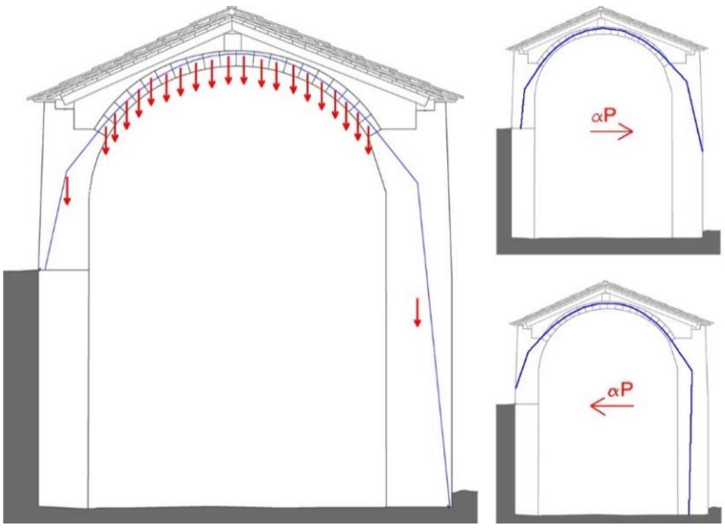


Figure 9: Thrust line in static Mery’s method (left) and in Heyman’s criterion, which was applied also including lateral loadings (right).

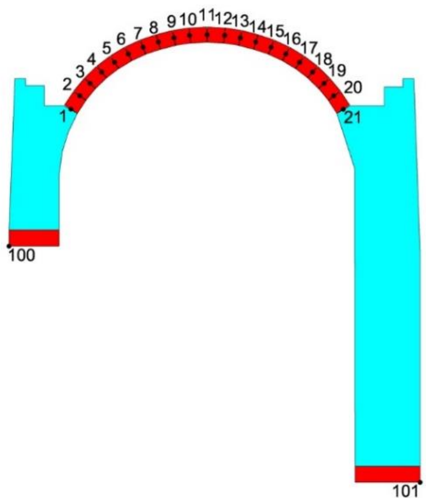


Figure 10: 2-dimensional vault TREMURI model.

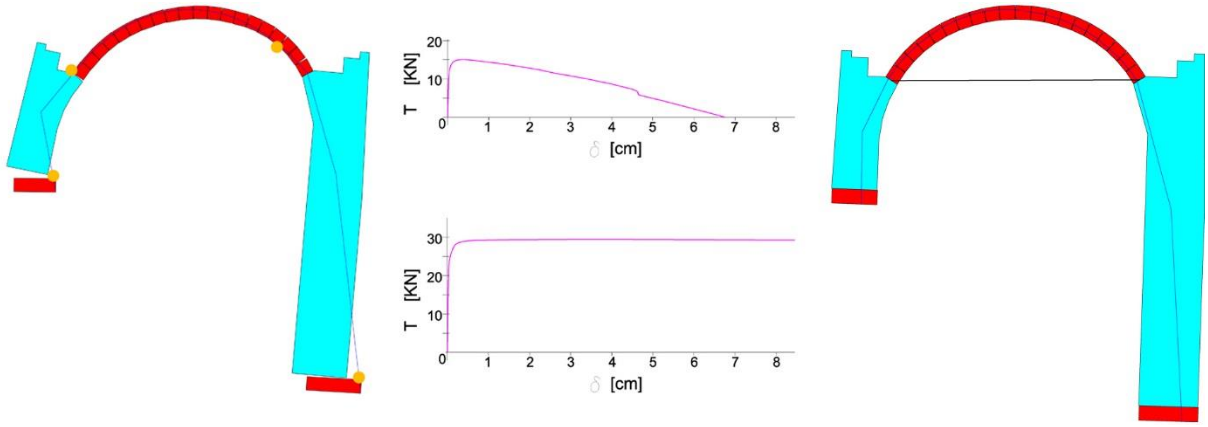
The results of nonlinear analysis show qualitatively that, after a specific value of the horizontal displacement, some partialization occurs in between the vault ashlars (macro-elements). The partialization develops at the extrados and the intrados of the vault, forming hinges that govern the development of the failure mechanism.

As it can be noticed from the diagram reported in Figure 11b (top), the evolution of the total horizontal thrust T for increasing lateral displacement δ shows a curve peak that corresponds to a critical multiplicative factor α, beyond which any further displacement tends to reduce the resistance due to second order effects. This curve defines the capacity of the structure and can be divided into:

- a first linear portion, which describes the elastic phase of the system, for which α is 0.105, similar to the value 0.12 determined by the graphical method;

- a increasing curved portion up to a peak and a curved downward, that describe the evolution with plastic deformation and effects of the second order, for which α is decreasing with maximum value of 0.18.

The effect of the introduction of a steel tie-rod (16 mm diameter) connecting the nodes at the vault haunches, was also evaluated by means of TREMURI (Figure 11c).
The associated diagram, in Figure 11b (bottom), shows that the introduction of the tie-rod provides a significant enhancement of the lateral capacity of the vault structural system.



(a) (b) (c)

Figure 11: (a) Reponse of the as-built vault system with indication of the hinges of the failure mechanism; (b) Force-displacement diagrams before (top) and after (bottom) the intervention; (c) vault model with tie-rod.

3.3 The arch

Between the nave and the presbytery there is an arched opening that overlooks three steps. This masonry arch (triumphal arch), like the previous vault, has not a homogeneous curved development, but has a polycentric conformation. The arch thickness is discretized to a maximum size of 19 cm, while the depth in which it develops is equal to 81 cm. Above this arch there is a portion of the wall shaped inclined to create the support of the roof pitches.

At first, we looked for the ideal thrust line. The model used is divided into ten equal segments (Figure 14a). The weight of each ashlar is equal to 735.6 N, in which is included a 20% increase to consider also the mortar weight. The weight of the portion of the wall above each ashlar is different and it depends on the specific gravity of the masonry. The external loads from the roof are:

- G, weight of the roof elements = 3564 N

- Snow + Wind + Q accidental = 3351 N

With the graphical method of Mery, we could draw the thrust line, which is always contained in the safety core of each ashlar. Then, we studied the arch response to lateral (seismic) loadings: we study graphically the limit conditions set in which the structural safety is guaranteed. In Figure 12 two different thrust lines are reported condiering both positive and negative loading directions. In the first case (top) α is equal to 0.34, while in the second (bottom) α is 0.25.

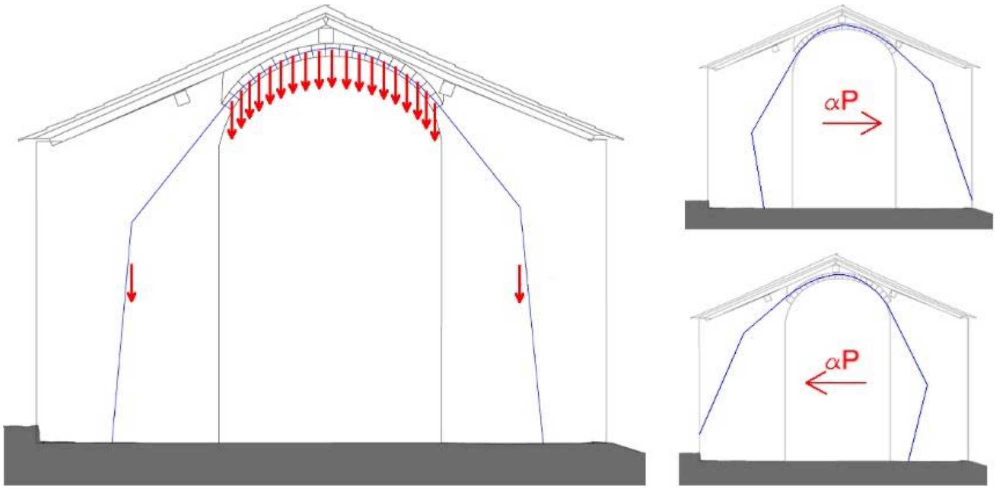


Figure 12: Thrust line in static Mery's method (left) and in Heyman's criterion, which was applied also including lateral loadings (right).

In order to evaluate the effects of the second order on this model and establish the critical value of α, the in-plane behavior of the whole system was modeled by means of the TREMURI program (Figure 13).

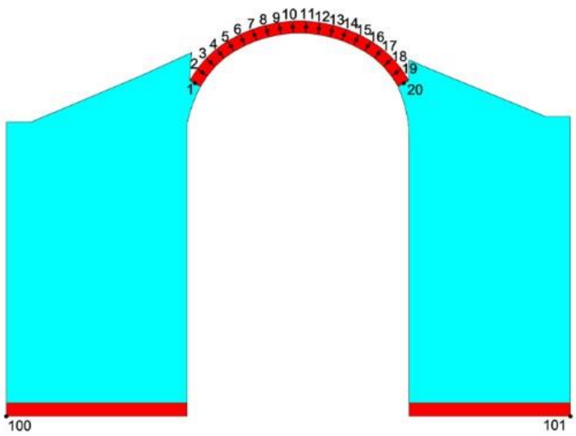
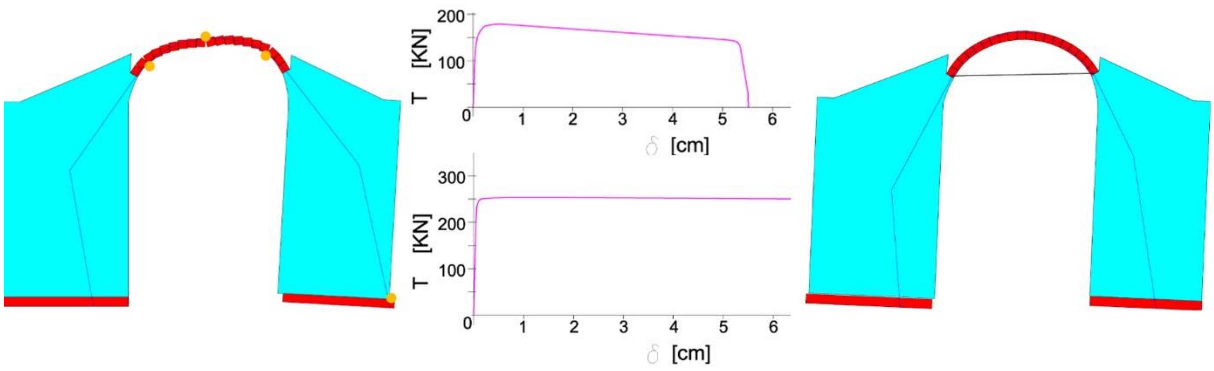


Figure 13: TREMURI arch model.

The results of non-linear analysis showed that, after a specific value of the horizontal displacement, some hinges appear to the extrados and the intrados of the arch (Figure 14a). The analysis showed that once the mechanism was activated second order effects reduced the lateral force for increasing lateral displacements. From the force-displacement diagram reported in Figure 14b (top), it can be noticed that the limit of the first linear-elastic phase corresponds to α = 0.19 which is then followed by a cracked phase with a peak value of α equal to 0.34.

Also in this case, the effects of the introduction of a steel tie-rod, which connects the nodes to the arch haunches (Figure 14c), was evaluated numrically. The force-diplacement diagram reported in Figure 14b (bottom) shows the improvement in the lateral response due to the considered strengthening intervention.



(a) (b) (c)

Figure 14: (a) Reponse of the as-built arch system with indication of the hinges of the failure mechanism; (b) Force-displacement diagrams before (top) and after (bottom) the intervention; (c) arch model with tie-ro

4 CONCLUSIONS

The whole analysis allowed us to know the building more deeply, not only in its material substance, but also in relation to techniques in which it was built. The structural study revealed that the medieval building techniques, typical of the area of the valley Trebbia, can be preserved by means of the introduction of simple strengthening devices.

As said before, testing materials directly was not possible and this aspect would have been an important step to complete the building knowledge and accurately define the structural analysis. Despite this, this study was an interesting example of application of different linear and nonlinear analysis techniques for the structural assessment.

In particular, the application of the TREMURI model to the analysis and simulation of structures including arches and vaults provided meaningful information and opens new fields of investigation for the structural analysis of cultural heritage buildings.

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